Published by Institute of Physics Publishing for SISSA/ISAS

Received: *June 26, 2003*
Accepted: *July 31, 2003*

Deformed superspace, $\mathcal{N} = 1/2$ supersymmetry & (non)renormalization theorems

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Abstract: We consider a deformed superspace in which the coordinates θ do not anticommute, but satisfy a Clifford algebra. We present results on the properties of $\mathcal{N} = 1/2$ supersymmetric theories of chiral superfields in deformed superspace, taking the Wess-Zumino model as the prototype. We prove new (non)renormalization theorems: the F-term is radiatively corrected and becomes indistinguishable from the D-term, while the $\overline{\text{F}}$-term is not renormalized. Supersymmetric vacua are critical points of the antiholomorphic superpotential. The vacuum energy is zero to all orders in perturbation theory. We illustrate these results with several examples.

Keywords: Renormalization Regularization and Renormalons, Superspaces, Non-Commutative Geometry.

JHEP07(2003)067

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http://jhep.sissa.it/archive/papers/jhep072003067/jhep072003067.pdf

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1. Introduction

It has been known that noncommutative geometry arises quite naturally in string theory [1]: by turning on a constant Kalb-Ramond B-field, Grassmann-even coordinates x^m are made noncommuting and obeying the Heisenberg algebra:

$$[x^m, x^n] = i\theta^{mn} \, . \tag{1.1}$$

In the Seiberg-Witten decoupling limit [2], noncommutative field theories arise on the worldvolume of D-branes. These field theories have revealed surprising features reminiscent of closed strings, such as UV-IR mixing [3] and nonlocal observables (open Wilson lines) [4], both of which turn out to be deeply intertwined [5]. Interplay between noncommutative field theories and string theories has been a rich and fruitful source for better understanding of both.

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A natural question is whether a noncommutative *super-geometry* can arise from string theory as well, where, instead of the Grassmann-even ones, the Grassmann-odd coordinates $\theta^\alpha(\alpha = 1, 2)$ are made noncommuting and obeying a Clifford algebra:

$$\left\{\theta^\alpha, \theta^\beta\right\} = C^{\alpha\beta}\,. \tag{1.2}$$

Recently, motivated partly by the development of a gauge theory - matrix model correspondence and its underlying string theory setup [6], an affirmative answer to the question was obtained: turning on Ramond-Ramond graviphoton field strength, the noncommutativity eq. (1.2) emerges again quite naturally from string theories [7, 8]. Its consequences are potentially far reaching, and the development calls for better understanding of field theories defined on noncommutative superspace eq. (1.2). Ground-breaking work in this direction appeared recently in [9]. To distinguish them from noncommutative supersymmetric field theories in which Grassmann-even coordinates are noncommuting, we will refer to the theories under consideration as *deformed supersymmetric field theories.*

In particular, we would like to consider (1.1) as the only deformation of the superspace; all other algebra on the superspace remains the same as usual once chiral coordinates are adopted [9]. This deformation preserves $\mathcal{N} = 1/2$ supersymmetry, as the supercharges Q_α, the generators of θ^α-translation, are conserved (see eq. (1.2)) while the $\overline{Q}_{\dot\alpha}$ are broken explicitly.

In this paper, we study how this deformation would modify quantum dynamics of supersymmetric field theories, paying particular attention to consequences of nonlocality in the superspace caused by eq. (1.2). We find quite a few surprises. We will build our analysis upon both superspace Feynman diagrammatics and symmetry considerations. For simplicity and clarity of presentation, we study primarily the deformation of the Wess-Zumino model. However, because our analysis is sufficiently general, the final results are equally valid for other theories. We will report further analysis in separate publications.

Our results are summarized as follows.

- The quantum effective action of the deformed Wess-Zumino model is expressed as

$$\begin{aligned}\Gamma[\Phi, \overline{\Phi}] = \sum_{n=0}^{\infty} \frac{1}{n!} \int & \mathrm{d}^4x_1 \cdots \mathrm{d}^4x_n \mathrm{d}^2\theta^2 \mathrm{d}^2\overline{\theta}^2 \times \\ & \times [F_1(x_1, \theta, \overline{\theta}) \cdots F_n(x_n, \theta, \overline{\theta}) G(x_1, \ldots, x_n; \overline{\theta\theta})]\,,\end{aligned} \tag{1.3}$$

  where $F_1, \ldots, F_n$ are functions involving background fields $\Phi, \overline{\Phi}$, possibly acted upon by $D_\alpha, \overline{D}_\alpha$, *and*, most significantly, by Q_α but not by $\overline{Q}_{\dot\alpha}$. The function $G(x_1, \ldots, x_n;$ $\overline{\theta\theta})$ is the result of superspace loop integrals and is translationally invariant. Again, because of the deformation, $(-\frac{1}{4}Q^2)$'s may act inside the loop integrals, and this results effectively in $\overline{\theta}^2$-dependence.

- The appearance of Q_α's is the only modification of the tree-level lagrangian once the deformation is considered. Its presence is anticipated by the observation that the

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star product

$$A(\theta) \star B(\theta) \equiv A(\theta) \exp\left(-\frac{1}{2}C^{ab}\frac{\overleftarrow{\partial}}{\partial\theta^\alpha}\frac{\overrightarrow{\partial}}{\partial\theta^\beta}\right) B(\theta)\,, \tag{1.4}$$

which implements the deformation in terms of the usual anticommuting coordinates, is re-expressible in terms of the chiral supercharge $Q_\alpha = \partial/\partial\theta^a$.

- Once Q_α's are tolerated in the tree-level lagrangian, one immediately finds that there is no real distinction between D-terms and F-terms in the quantum effective action: $(-\frac{1}{4}Q^2)$ acting inside loop integrals generates $\overline{\theta}^2$-dependence, so the $\mathrm{d}^2\overline{\theta}$ integral can yield a F-term. Consequently, once the deformation is made, not only the D-term but also the F-term is renormalized. In fact, the two become indistinguishable.

- The antiholomorphic $\overline{\mathrm{F}}$-term is not renormalized. This is because, even if Q_α's are present, the effective action $\Gamma(\Phi, \overline{\Phi})$ cannot yield any $\overline{\mathrm{F}}$-term by the $\mathrm{d}^2\theta$ integration. This $\mathcal{N} = 1/2$ nonrenormalization theorem, which we will prove in this work in generality, is indeed the prerequisite for the antichiral ring structure to survive at the quantum level.

- The vacua $|\mathrm{vac}\rangle, \langle\mathrm{vac}|$ that preserve the $\mathcal{N} = 1/2$ supersymmetry are characterized by a set of critical points of the antiholomorphic superpotential, $\overline{W}\,'(\overline{A}) = 0$.

- The vacuum energy is not renormalized: the $\mathcal{N} = 1/2$ supersymmetric vacuum has vanishing energy density and is stable against radiative corrections. It then follows from the supersymmetry algebra that

$$\langle\mathrm{vac}|Q_\alpha = 0 \qquad \text{and} \qquad Q_\alpha|\mathrm{vac}\rangle = 0\,. \tag{1.5}$$

  This is another prerequisite for existence of the antichiral ring structure in $\mathcal{N} = 1/2$ supersymmetry.

This paper is organized as follows. In section 2, we recapitulate the deformation of supersymmetric field theory and derive superspace Feynman rules. In section 3, we present the $\mathcal{N} = 1/2$ (non)renormalization theorems: (1) the F-term is renormalized, but the $\overline{\mathrm{F}}$-term is not; (2) the vacuum energy is zero to all orders in perturbation theory. We report the general structure and superspace expression of the quantum effective action. In section 4, we illustrate this result by computing several lower-order Feynman diagrams. In section 5, to illustrate the power of the new (non)renormalization theorems, we show the vanishing of the vacuum energy explicitly, using various (pseudo)symmetries. We explain how this vanishing preserves the antichiral ring. We show that supersymmetric vacua are critical points of the antiholomorphic superpotential. In section 6, we discuss two intriguing observations concerning the general structure of these theories as well as phenomenological prospects.

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2. Deformed supersymmetric field theory

2.1 Setup

We begin by recapitulating relevant aspects of the deformation [9] of supersymmetric field theories [10, 11] and by setting up the deformed Wess-Zumino model in a form suitable for our analysis.

Take $\mathcal{N} = 1$ superspace $\mathcal{S}$ in *euclidean* four dimensions. Choose the 'chiral basis' of the superspace coordinates $z^A = (y^m, \theta^\alpha, \overline{\theta}^{\dot{\alpha}})$, where $y^m \equiv (x^m + i\theta\sigma^m\overline{\theta})$ are the four real Grassmann-even coordinates, and $\theta^\alpha, \overline{\theta}^{\dot{\alpha}}$ are two independent Weyl Grassmann-odd coordinates. In the coordinates adopted, the superspace derivatives and supersymmetry generators are given by

$$D_\alpha = +\frac{\partial}{\partial\theta^\alpha} + 2i\sigma^m_{\alpha\dot{\alpha}}\overline{\theta}^{\dot{\alpha}}\frac{\partial}{\partial y^m}\,, \qquad \overline{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\overline{\theta}^{\dot{\alpha}}}\,, \tag{2.1}$$

$$\overline{Q}_{\dot{\alpha}} = -\frac{\partial}{\partial\overline{\theta}^{\dot{\alpha}}} + 2i\theta^\alpha\sigma^m_{\alpha\dot{\alpha}}\frac{\partial}{\partial y^m}\,, \qquad Q_\alpha = +\frac{\partial}{\partial\theta^\alpha}\,. \tag{2.2}$$

Chiral and antichiral superfields $\Phi, \overline{\Phi}$ are defined by $\overline{D}_{\dot{\alpha}}\Phi = 0$ and $D_\alpha\overline{\Phi} = 0$, respectively. Their expansion in component fields is

$$\begin{aligned}\Phi(y,\theta) &= A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y)\,;\\ \overline{\Phi}(\overline{y},\overline{\theta}) &= \overline{A}(\overline{y}) + \sqrt{2}\overline{\theta\psi}(\overline{y}) + \overline{\theta\theta}\overline{F}(\overline{y})\,.\end{aligned}$$

The deformed theory may then be defined as follows: in the chiral coordinates adopted, multiply superfields via the $\star$-product eq. (1.4). As demonstrated in [9], the $\star$-product of chiral superfields is again a chiral superfield; likewise, the $\star$-product of antichiral superfields is again an antichiral superfield. Consequently, the deformation of a supersymmetric field theory is defined by replacing all superfield multiplications by $\star$-product multiplcations. Thus, for the Wess-Zumino model, the deformation leads to the lagrangian density

$$L_{\mathrm{WZ-def}} = L_D + L_F + L_{\overline{F}}\,, \tag{2.3}$$

where

$$L_D = \Big[\Phi \star \overline{\Phi}\Big]_{\theta^2\overline{\theta}^2} = \overline{A}\Box A - i\overline{\psi}\overline{\sigma}^m\partial_m\psi + \overline{F}F \tag{2.4}$$

$$\begin{aligned}L_F &= \Big[\frac{1}{2}m\Phi\star\Phi + \frac{g}{3}\Phi\star\Phi\star\Phi\Big]_{\theta^2}\\ &= m\Big(AF - \frac{1}{2}\psi\psi\Big) + g\Big(AAF - A\psi\psi\Big) - \frac{g}{3}|C|FFF \end{aligned}\tag{2.5}$$

$$\begin{aligned}L_{\overline{F}} &= \Big[\frac{1}{2}\overline{m}\overline{\Phi}\star\overline{\Phi} + \frac{\overline{g}}{3}\overline{\Phi}\star\overline{\Phi}\star\overline{\Phi}\Big]_{\overline{\theta}^2}\\ &= \overline{m}\Big(\overline{AF} - \frac{1}{2}\overline{\psi\psi}\Big) + \overline{g}\Big(\overline{AAF} - \overline{A\psi\psi}\Big)\,.\end{aligned}\tag{2.6}$$

We see that the sole effect of the deformation is that the F-term receives a new contribution proportional to $|C|$, the determinant of $C^{\alpha\beta}$. Notice that this new term can be expressed

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in terms of ordinary products as[1]

$$\Delta_{\rm def} L_F = -\frac{g}{3}|C| \left[\left(\left(-\frac{1}{4} Q^2 \right) \Phi \right)^2 \Phi \right]_{\theta^2} . \tag{2.7}$$

Consequently, one can view the deformed Wess-Zumino model as the ordinary Wess-Zumino model, where superfield multiplication is standard, with a new addition eq. (2.7) to the F-term. That is, the deformed Wess-Zumino model is definable by the lagrangian

$$L_{\rm WZ-def} = \left[\Phi\overline{\Phi}\right]_{\theta^2\overline{\theta}^2} + \left[\frac{m}{2}\Phi\Phi + \frac{g}{3}\Phi\Phi\Phi\right]_{\theta^2} + \left[\frac{\overline{m}}{2}\overline{\Phi\Phi} + \frac{\overline{g}}{3}\overline{\Phi\Phi\Phi}\right]_{\overline{\theta}^2} + \Delta_{\rm def} L_F . \tag{2.8}$$

In components, the lagrangian is

$$\begin{aligned} L_{\rm WZ-def} =\ & \overline{A}\Box A - i\overline{\psi}\overline{\sigma}^m \partial_m \psi + \overline{F}F + m\left(AF - \frac{1}{2}\psi\psi\right) + \overline{m}\left(\overline{AF} - \frac{1}{2}\overline{\psi\psi}\right) + \\ & + g(AAF - A\psi\psi) + \overline{g}(\overline{AAF} - \overline{A\psi\psi}) - \frac{g}{3}|C|FFF . \end{aligned} \tag{2.9}$$

Adding the last term in eqs. (2.8) and (2.9) renders the theory quite novel. In ordinary Wess-Zumino model, because of $\mathcal{N}=1$ supersymmetry, one is not allowed to introduce Q_α and $\overline{Q}_{\dot\alpha}$ in the lagrangian. With the deformation, however, the $\overline{Q}_{\dot\alpha}$-part of the supersymmetry is broken explicitly, so here Q_α's are allowed. Conversely, if Q_α's are present in the lagrangian, the antichiral part of the $\mathcal{N}=1$ supersymmetry (associated with translation of $\overline{\theta}$) is broken, and the theory preserves $\mathcal{N}=1/2$ supersymmetry only. In fact, as mentioned above, the definition of $\star$-product is expressible entirely in terms of Q_α's

$$\Phi_1(y,\theta) \star \Phi_2(y,\theta) = \Phi_1(y,\theta) \exp\left(-\frac{1}{2} C^{\alpha\beta} \overleftarrow{Q_\alpha} \overrightarrow{Q_\beta} \right) \Phi_2(y,\theta) \tag{2.10}$$

at fixed y in the chiral coordinates. Therefore, it is not surprising that, when recast in ordinary superspace formulation, the chiral supercharges Q_α show up in the lagrangian. Consequences of such explicit $\mathcal{N}=1/2$ supersymmetry breaking are quite interesting, as we demonstrate in detail in later sections.

2.2 Deformed Feynman rules

We now present Feynman rules for the deformed Wess-Zumino model. They are derived by straightforward application of the standard method [10]–[12]. We present the rules both in component form and in superfield form. We also summarize various identities utilized for later computations.

The superspace Feynman rules are:

1. Use the so-called GRS propagators [13] for internal lines. They are given as follows:

$$\begin{aligned} \left\langle \Phi(z)\overline{\Phi}(z') \right\rangle &= \frac{i}{\Box - m\overline{m}} \delta^8(z-z') , \\ \left\langle \Phi(z)\Phi(z') \right\rangle &= \frac{-i\overline{m}}{(\Box - m\overline{m})} \frac{1}{\Box} \left(-\frac{1}{4} D_z^2 \right) \delta^8(z-z') , \\ \left\langle \overline{\Phi}(z)\overline{\Phi}(z') \right\rangle &= \frac{-im}{(\Box - m\overline{m})} \frac{1}{\Box} \left(-\frac{1}{4} \overline{D}_z^2 \right) \delta^8(z-z') , \end{aligned} \tag{2.11}$$

[1] Recall that $Q^2\Psi$ is also a (anti)chiral superfield if Ψ is so. This follows trivially from the fact $\{Q, D\} = \{Q, \overline{D}\} = 0$.

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where $\delta^8(z-z') = \delta^4(x-x')\delta^2(\theta-\theta')\delta^2(\overline{\theta}-\overline{\theta}')$. For later reference, we also record the propagators in component form

$$\begin{aligned}
\langle A(x)\overline{A}(x')\rangle &= \frac{i}{\Box - m\overline{m}}\delta^4(x-x') \qquad \langle F(x)\overline{F}(x')\rangle = \frac{i\,\Box}{\Box - m\overline{m}}\delta^4(x-x') \\
\langle A(x)F(x')\rangle &= \frac{-i\,\overline{m}}{\Box - m\overline{m}}\delta^4(x-x') \qquad \langle \overline{A}(x)\overline{F}(x')\rangle = \frac{-i\,m}{\Box - m\overline{m}}\delta^4(x-x') \\
\langle \psi_\alpha(x)\psi^\beta(x')\rangle &= \delta_\alpha^\beta \frac{i\,\overline{m}}{\Box - m\overline{m}}\delta^4(x-x') \qquad \langle \overline{\psi}^{\dot{\alpha}}(x)\overline{\psi}_{\dot{\beta}}(x')\rangle = \delta_{\dot{\beta}}^{\dot{\alpha}} \frac{i\,m}{\Box - m\overline{m}}\delta^4(x-x') \\
\langle \psi_\alpha(x)\overline{\psi}_{\dot{\beta}}(x')\rangle &= \frac{\sigma^m_{\alpha\dot{\beta}}\partial^x_m}{\Box - m\overline{m}}\delta^4(x-x')\,,
\end{aligned} \tag{2.12}$$

where $\Box = \partial_m \partial_m$.

2. There are two chiral vertices, Φ^3, $(-\frac{1}{4}Q^2\Phi)^2\Phi$; and one antichiral vertex, $\overline{\Phi}^3$. Associated with every chiral vertex carrying n internal lines, $(n-1)$ factors of $\left(-\frac{1}{4}\overline{D}^2\right)$ act on some arbitrary $(n-1)$ internal lines. Likewise, associated with every antichiral vertex carrying n internal lines, $(n-1)$ factors of $\left(-\frac{1}{4}D^2\right)$ act on some arbitrary $(n-1)$ internal lines. All external lines arise without any of these factors.

3. For the new vertex $\left(-\frac{1}{4}Q^2\Phi\right)^2\Phi$, two factors of $\left(-\frac{1}{4}Q^2\right)$ are attached to an arbitrary two of the three (external or internal) lines.

4. Associated to every vertex, multiply the appropriate factor of $\frac{1}{3}g$, $\frac{1}{3}\overline{g}$ or $-\frac{1}{3}g|C|$, and perform the superspace integral $\int \mathrm{d}^8 z$.

5. Compute the standard combinatoric factors for a given theory.

Notice that only rule 3 is new for the deformed theory. As such, the deformed Feynman rules are quite general, and extend straightforwardly to other field theories.

Recall that, for a chiral superfield Φ, $\Xi \equiv (-\frac{1}{4}Q^2)\Phi$ is also a chiral superfield. Therefore, the deformed superspace Feynman rules are exactly the same as the ordinary superspace Feynman rules if we treat Ξ as an independent chiral superfield and Wick-contract Ξ with the propagators

$$\begin{aligned}
\langle \Xi(z)\Phi(z')\rangle &= \left(-\frac{Q^2}{4}\right)\langle \Phi(z)\Phi(z')\rangle \\
\langle \Xi(z)\overline{\Phi}(z')\rangle &= \left(-\frac{Q^2}{4}\right)\langle \Phi(z)\overline{\Phi}(z')\rangle\,,
\end{aligned} \tag{2.13}$$

and similarly for antichiral counterparts.

3. (Non)-renormalization theorems

We now compute the one-particle-irreducible effective action. We will keep both $m, \overline{m}$ nonzero so that the effective action is well defined in the infrared.

Before presenting the general structure of the effective action in the deformed theory, it will be useful to recollect the standard non-renormalization theorems of supersymmetric field theories as summarized for example in [12]:

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- **Theorem 1.** *Each term in the effective action is expressible as a superspace integral over a single* $\mathrm{d}^2\theta\mathrm{d}^2\overline{\theta}$.

- **Theorem 2.** *The general structure of the effective action is given as*

$$\Gamma[\Phi,\overline{\Phi}] = \sum_n \int \prod_{j=1}^{n} \mathrm{d}^4 x_j \int \mathrm{d}^2\theta \mathrm{d}^2\overline{\theta}\, G_n(x_1,\ldots,x_n) F_1(x_1,\theta)\cdots F_n(x_n,\theta)\,.$$

  where $G_n(x_1,\ldots,x_n)$ *are translation-invariant functions on Grassmann-even coordinates and* $F(x,\theta,\overline{\theta})$ *are* local *operators of* $\Phi,\overline{\Phi}$ *and their covariant derivatives:*

$$F(x,\theta,\overline{\theta}) = F(\Phi,\overline{\Phi},D\Phi,\overline{D\Phi},\ldots)\,.$$

The above theorems, especially Theorem 2, lead immediately to the following results: (1) energy density of supersymmetric vacuum is zero because, in this case, there are no $F(x,\theta,\overline{\theta})$ field insertions in the effective action, so the $\int \mathrm{d}^2\theta\mathrm{d}^2\overline{\theta}$ integral gives zero; (2) the holomorphic and antiholomorphic parts are not renormalized. The reason is that to get holomorphic part one needs to integrate out $\int \mathrm{d}^2\overline{\theta}$. However, as there is no $\Box^{-1}$ in the effective action, one cannot do that by combining it with the D^2 operator. A similar argument holds for the antiholomorphic part.

Now in our deformed theory, Theorem 1 is not modified. The proof goes exactly the same as the ordinary theory. However, Theorem 2 is modified crucially by the new vertex with insertion of the operator $-\frac{1}{4}Q^2$. It follows from the Feynman rules in the previous section that this operator affects loop integrals in a way similar to $-\frac{1}{4}D^2$ and $-\frac{1}{4}\overline{D}^2$. Thus we derive the following new theorem.

- **Theorem 3.** [after deformation]: *The general structure of the effective action is given as*

$$\Gamma[\Phi,\overline{\Phi}] = \sum_n \int \prod_{j=1}^{n} \mathrm{d}^4 x_j \int \mathrm{d}^2\theta \mathrm{d}^2\overline{\theta}\, G_n(x_1,\ldots,x_n;\overline{\theta\theta}) F_1(x_1,\theta,\overline{\theta})\cdots F_n(x_n,\theta,\overline{\theta})\,, \tag{3.1}$$

  where $G_n(x_1,\ldots,x_n;\overline{\theta\theta})$ *are translation-invariant functions on Grassmann-even coordinates* and *possible insertion of* $\overline{\theta\theta}$, *while* $F(x,\theta,\overline{\theta})$ *are local operators of* $\Phi,\overline{\Phi}$, *their covariant derivatives,* and *the action of the chiral supercharge* Q*:*

$$F(x,\theta,\overline{\theta}) = F(\Phi,\overline{\Phi},D\Phi,\overline{D\Phi},Q\Phi,Q\overline{\Phi}\ldots)\,.$$

We will present later explicit computations and symmetry arguments to substantiate the general structure of the effective action as claimed, but the crux of the new theorem stems from insertion of Q_α and its effects.

Using the modified theorem 3, we are now able to derive the following results: (1) energy density of supersymmetric vacuum is still zero. Although the $\overline{\theta\theta}$-dependence $G(x_1,\ldots,x_n;\overline{\theta\theta})$ would be able to render the $\int \mathrm{d}^2\overline{\theta}$ integral nonzero, in the absence of any $F(x,\theta,\overline{\theta})$ insertions, the $\int \mathrm{d}^2\theta$ integral still vanishes; (2) The antiholomorphic part is still not renor-

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malized, because the $\int d^2\theta$ are not absorbable, for the same reason as in the ordinary Wess-Zumino model; (3) However, the holomorphic part *is renormalized.* The reason is that now we have the $\overline{\theta}\overline{\theta}$ insertion from $G(x_1,\ldots,x_n;\overline{\theta}\overline{\theta})$, which can absorb the $\int d^2\overline{\theta}$ integral. Because of this, the D-terms with pure chiral fields and holomorphic F-terms are not distinguishable, and in fact both D-terms and F-terms are unified in $\mathcal{N}=1/2$ supersymmetry. We emphasize that this is the feature that was not evident from the classical consideration [9], but was revealed only after full quantum effects are taken into account.

4. Illustration by diagrams

In this section we outline the computation of a few Feynman diagrams that contribute to the effective action with a single factor of $|C|$. These examples illustrate the diagrammatic consequences of the new vertex, $(-\frac{1}{4}Q^2\Phi)^2\Phi$. We will see how the extra factors of Q^2 can appear sometimes on the external lines, sometimes as factors of $\overline{\theta}\overline{\theta}$, and sometimes disappear altogether.

We begin with the simplest such diagram, $\langle FF\rangle$ at one loop, which generates the term $\overline{\theta}\overline{\theta}(Q^2\Phi)\Phi$ in the effective action. The next example is similar, a one-loop diagram for $\langle \overline{A}FF\rangle$ that generates $\overline{\theta}\overline{\theta}(Q^2\Phi)\Phi\overline{\Phi}$. The last few examples show how to generate terms without $\overline{\theta}\overline{\theta}$ or Q^2. We also discover nonholomorphic corrections to the couplings of seemingly holomorphic F-terms.

The principal tools in these calculations are integration by parts and the identities listed below for differential operators acting on superspace delta functions.

$$\epsilon^{\alpha\beta}\left(\sigma^m_{\alpha\dot\alpha}\overline{\theta}^{\dot\alpha}\partial_m\right)\left(\sigma^n_{\beta\dot\beta}\overline{\theta}^{\dot\beta}\partial_n\right) = -\overline{\theta}\overline{\theta}\Box \qquad \left(\theta^\alpha\sigma^m_{\alpha\dot\alpha}\partial_m\right)\left(\theta^\beta\sigma^n_{\beta\dot\beta}\partial_n\right)\epsilon^{\dot\alpha\dot\beta} = +\theta\theta\Box \tag{4.1}$$

$$\begin{aligned}\left(\frac{-D^2}{4}\right)\left(\frac{-\overline{D}^2}{4}\right)D^2 &= D^2\Box \quad D^2D'^2\delta^8\left(z-z'\right) = D'^2D^2\delta^8\left(z-z'\right)\\ \left(\frac{-\overline{D}^2}{4}\right)\left(\frac{-D^2}{4}\right)\overline{D}^2 &= \overline{D}^2\Box \quad D^2\overline{D}'^2\delta^8\left(z-z'\right) = \overline{D}'^2D^2\delta^8\left(z-z'\right)\end{aligned} \tag{4.2}$$

$$\begin{aligned}\left(\frac{-D^2}{4}\right)\left(\frac{-\overline{D}^2}{4}\right)\delta^4\left(\theta-\theta'\right)\Big|_{\theta=\theta'} &= 1 \qquad D_\alpha\delta^8\left(z-z'\right) = -D'_\alpha\delta^8\left(z-z'\right)\\ \left(\frac{-Q^2}{4}\right)\left(\frac{-D^2}{4}\right) &= \overline{\theta}\overline{\theta}\Box\left(\frac{1}{4}\epsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta^\beta}\right)\\ \left(\frac{-Q^2}{4}\right)\Phi\left(y,\theta\right) &= F\left(y\right)\\ \left(\frac{-Q^2}{4}\right)\left(\frac{-\overline{D}^2}{4}\right)\delta^4\left(\theta-\theta'\right)|_{\theta=\theta'} &= 1\\ \left(\frac{-Q^2}{4}\right)\left(\frac{-D^2}{4}\right)\left(\frac{-\overline{D}^2}{4}\right)\delta^4\left(\theta-\theta'\right)|_{\theta=\theta'} &= \overline{\theta}\overline{\theta}\Box\,.\end{aligned} \tag{4.3}$$

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Figure 1: The one loop correction to $\langle FF\rangle$. (a) the component diagram (b) the corresponding superfield diagram.

4.1 A term with Q^2: two-point function

The $\langle FF\rangle$ correction at one loop is the simplest new contribution to the effective action with Q^2 acting on the external line. See figure 1. The term in the effective action is of the form $\overline{\theta\theta}(Q^2\Phi)\Phi$.

In component form, we have

$$6\left(-\frac{g}{3}|C|\right) g \int \mathrm{d}^4x_1 \mathrm{d}^4x_2 F(x_1)F(x_2)\left[\frac{-i\overline{m}}{\Box - m\overline{m}}\delta(x_1-x_2)\right]^2 =$$

$$= 2g^2|C|\overline{m}^2 \int \mathrm{d}^4x_1 \mathrm{d}^4x_2 F(x_1)F(x_2)\left[\frac{1}{\Box - m\overline{m}}\delta(x_1-x_2)\right]^2 , \qquad (4.4)$$

where 6 is the symmetry factor. The integral diverges logarithmically.

In superfield form, we have

$$18\left(-\frac{g}{3}|C|\right)\frac{g}{3}\int \mathrm{d}^4x_1\mathrm{d}^4x_2\mathrm{d}^4\theta_1\mathrm{d}^4\theta_2\left(\frac{-Q_1^2}{4}\Phi(x_1,\theta_1)\right)\Phi\left(x_2,\theta_2\right)\times$$

$$\times\left[\frac{-Q_1^2}{4}\frac{-\overline{D}_1^2}{4}\frac{i\overline{m}D_1^2}{4\Box(\Box - m\overline{m})}\frac{-\overline{D}_2^2}{4}\delta^8(z_1-z_2)\right]\left[\frac{i\overline{m}D_1^2}{4\Box(\Box - m\overline{m})}\delta^8(z_1-z_2)\right],$$

where 18 is the symmetry factor.

The calculation in superfield form proceeds as follows. In the first bracketed factor, replace $\frac{-\overline{D}_2^2}{4}$ by $\frac{-\overline{D}_1^2}{4}$ since it is acting directly on the delta function. This is a manipulation we shall use often. It is valid because all preceding operators can be stripped by integration by parts, the substitution made by the last identity of eq. (4.2), and then all preceding operators replaced.

Then apply another identity from eq. (4.2) to this first bracketed factor, to replace the product $\frac{-\overline{D}_1^2}{4}\frac{\overline{D}_1^2}{4\Box}\frac{-\overline{D}_1^2}{4}$ by $\frac{-\overline{D}_1^2}{4}$. Integrate by parts under $\frac{-Q_1^2}{4}$ and $\frac{-\overline{D}_1^2}{4}$, noting that $\frac{-Q_1^2}{4}\Phi$ is a chiral field, to get

$$-2g^2|C|\int \mathrm{d}^4x_1\mathrm{d}^4x_2\mathrm{d}^4\theta_1\mathrm{d}^4\theta_2\left(\frac{-Q_1^2}{4}\Phi(x_1,\theta_1)\right)\Phi(x_2,\theta_2)\times$$

$$\times\left[\frac{i\overline{m}}{4(\Box - m\overline{m})}\delta^8(z_1-z_2)\right]\left[\frac{-Q_1^2}{4}\frac{i\overline{D}_1^2\overline{m}D_1^2}{4\Box(\Box - m\overline{m})}\delta^8(z_1-z_2)\right].$$

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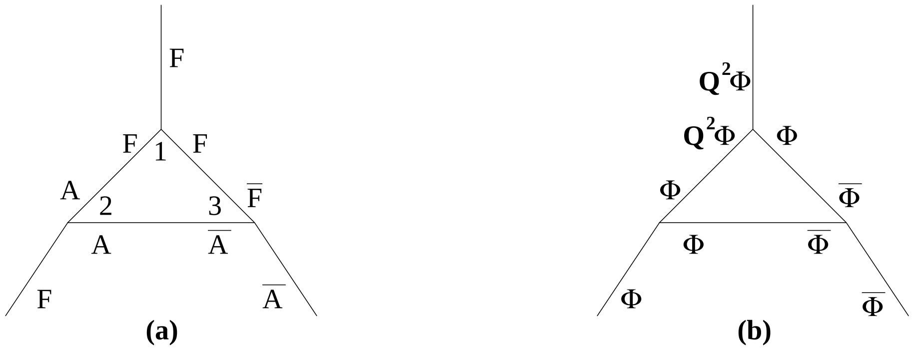

Figure 2: The one loop correction to $\langle FF\overline{A}\rangle$. (a) the component diagram (b) the corresponding superfield diagram.

Finally, use the symmetry properties of delta functions, and commutation of the operators, to change the indices of the differential operators in the last factor all to 2. Note that this comes at the cost of exchanging the order of $\overline{D}^2$ and D^2, since we can only exercise this symmetry on the operator directly in front of the delta function. Then apply eq. (4.3) to find

$$2g^2|C|\overline{m}^2 \int d^4x_1 d^4x_2 d^4\theta_1 \left(\frac{-Q_1^2}{4}\Phi(x_1,\theta_1)\right)\Phi(x_2,\theta_1)(\overline{\theta}_1\overline{\theta}_1)\left[\frac{1}{\Box - m\overline{m}}\delta^4(x_1-x_2)\right]^2,$$

which reproduces the expression eq. (4.4) in component fields.

4.2 A term with Q^2: three-point function

There are two superfield diagrams at one loop with the new vertex $(-\frac{1}{4}Q^2\Phi)^2\Phi$ that generate three-point functions with Q^2 acting on external lines. One of these combines two Φ^3 vertices and one $(Q^2\Phi)^2\Phi$ vertex, generating a term $\overline{\theta\theta}(Q^2\Phi)\Phi^2$ in the effective action. The other, shown in Figure 2, generates a term $\overline{\theta\theta}(Q^2\Phi)\Phi\overline{\Phi}$ in the effective action. The calculations are very similar; here we present the second. Note that while the contribution from the two-point function in the previous subsection could have passed for an F-term, this one cannot, because it includes an antichiral field.

The corresponding diagram in component fields is shown in Figure 2. The integral form is

$$\begin{aligned}
&3(2^3)\left(-\frac{g}{3}|C|\right)g\overline{g}\int d^4x_1 d^4x_2 d^4x_3 F(x_1)F(x_2)\overline{A}(x_3)\times \\
&\times\left[\frac{-i\overline{m}}{\Box - m\overline{m}}\delta^4(x_1-x_2)\right]\left[\frac{i\Box}{\Box - m\overline{m}}\delta^4(x_1-x_3)\right]\left[\frac{i}{\Box - m\overline{m}}\delta^4(x_2-x_3)\right] = \\
&\quad = -8ig^2\overline{g}|C|\overline{m}\int d^4x_1 d^4x_2 d^4x_3 F(x_1)F(x_2)\overline{A}(x_3)\times \\
&\qquad\times\left[\frac{1}{\Box - m\overline{m}}\delta^4(x_1-x_2)\right]\left[\frac{\Box}{\Box - m\overline{m}}\delta^4(x_1-x_3)\right]\left[\frac{1}{\Box - m\overline{m}}\delta^4(x_2-x_3)\right]. \quad (4.5)
\end{aligned}$$

Again the contribution diverges logarithmically.

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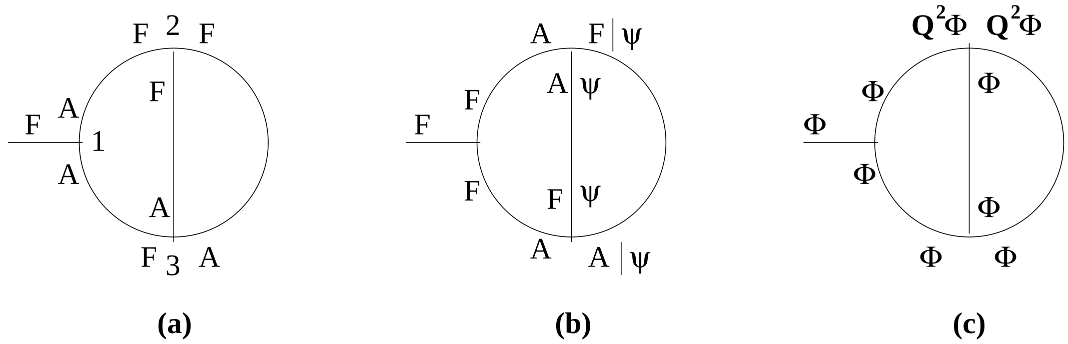

Figure 3: The tadpole diagram $\langle F\rangle$. There are two distinguished cases: (a) all three lines of F^3 vertex are contracted; (b) one line of F^3 vertex is not contracted. However, for case (b), bosonic and fermionic loop contributions cancel each other. (c) is the corresponding superfield diagram of case (a).

The superfield calculation is much like the previous one. The expression is

$$\begin{aligned}
&6^3\left(-\frac{g}{3}|C|\right)\frac{g}{3}\frac{\overline{g}}{3}\int \mathrm{d}^4x_1\mathrm{d}^4x_2\mathrm{d}^4x_3\mathrm{d}^4\theta_1\mathrm{d}^4\theta_2\mathrm{d}^4\theta_3\left(\frac{-Q_1^2}{4}\Phi(x_1,\theta_1)\right)\Phi(x_2,\theta_2)\overline{\Phi}(x_3,\theta_3)\times\\
&\times\left[\frac{-Q_1^2}{4}\frac{-\overline{D}_1^2}{4}\frac{i\overline{m}D_1^2}{4\Box(\Box-m\overline{m})}\frac{-\overline{D}_2^2}{4}\delta^8(z_1-z_2)\right]\left[\frac{-D_3^2}{4}\frac{i}{\Box-m\overline{m}}\delta^8(z_1-z_3)\right]\times\\
&\times\left[\frac{i}{\Box-m\overline{m}}\delta^8(z_2-z_3)\right].
\end{aligned}$$

Upon application of integration by parts and the identities in eqs. (4.2)–(4.3), it reduces to

$$\begin{aligned}
&-8ig^2\overline{g}|C|\overline{m}\int \mathrm{d}^4x_1\mathrm{d}^4x_2\mathrm{d}^4x_3\mathrm{d}^4\theta(\overline{\theta\theta})\left(\frac{-Q_1^2}{4}\Phi(x_1,\theta)\right)\Phi(x_2,\theta)\overline{\Phi}(x_3,\theta)\times\\
&\times\left[\frac{1}{\Box-m\overline{m}}\delta^8(z_1-z_2)\right]\left[\frac{\Box}{\Box-m\overline{m}}\delta^4(x_1-x_3)\right]\left[\frac{1}{\Box-m\overline{m}}\delta^4(x_2-x_3)\right],
\end{aligned}$$

which is the same result as eq. (4.5).

4.3 A term in which Q^2's disappear

There is a two-loop tadpole for F, shown in Figure 3, that corresponds to a $\overline{\theta\theta}\Phi$ term in the effective action. As shown in the figure, there are three contributions at this order (or two in terms of superfields), but supersymmetry cancels two of them (or one in terms of superfields). We sketch the computation to observe how the factors of Q^2 disappear in the result.

The integral in component fields is

$$\begin{aligned}
&24\left(-\frac{g}{3}|C|\right)\frac{1}{2}g^2\int \mathrm{d}^4x_1\mathrm{d}^4x_2\mathrm{d}^4x_3F(x_1)\left[\frac{-i\overline{m}}{\Box-m\overline{m}}\delta^4(x_1-x_2)\right]\times\\
&\times\left[\frac{-i\overline{m}}{\Box-m\overline{m}}\delta^4(x_1-x_3)\right]\left[\frac{-i\overline{m}}{\Box-m\overline{m}}\delta^4(x_2-x_3)\right]\left[\frac{-i\overline{m}}{\Box-m\overline{m}}\delta^4(x_2-x_3)\right]=
\end{aligned}$$

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$$= -4g^3|C|\overline{m}^4 \int \mathrm{d}^4x_1\mathrm{d}^4x_2\mathrm{d}^4x_3 F(x_1)\left[\frac{1}{\Box - m\overline{m}}\delta^4(x_1-x_2)\right] \times$$
$$\times \left[\frac{1}{\Box - m\overline{m}}\delta^4(x_1-x_3)\right]\left[\frac{1}{\Box - m\overline{m}}\delta^4(x_2-x_3)\right]\left[\frac{1}{\Box - m\overline{m}}\delta^4(x_2-x_3)\right]. \quad (4.6)$$

The superfield integral is

$$72\left(-\frac{g}{3}|C|\right)\frac{1}{2}\left(\frac{g}{3}\right)^2\int\prod_{a=1}^{3}\mathrm{d}^4x_a\mathrm{d}^4\theta_a\left[\frac{-\overline{D}_3^2}{4}\frac{i\overline{m}D_1^2}{4\Box(\Box - m\overline{m})}\delta^8(z_3-z_1)\right] \times$$
$$\times \left[\frac{i\overline{m}D_2^2}{4\Box(\Box - m\overline{m})}\delta^8(z_2-z_3)\right]\left[\frac{-Q_2^2}{4}\frac{-\overline{D}_2^2}{4}\frac{i\overline{m}D_2^2}{4\Box(\Box - m\overline{m})}\frac{-\overline{D}_1^2}{4}\delta^8(z_2-z_1)\right] \times$$
$$\times \left[\frac{-Q_2^2}{4}\frac{-\overline{D}_2^2}{4}\frac{i\overline{m}D_2^2}{4\Box(\Box - m\overline{m})}\frac{-\overline{D}_3^2}{4}\delta^8(z_2-z_3)\right].$$

Manipulations similar to those in the previous calculations bring the expression into the form

$$-4g^3|C|\overline{m}^4\int\prod_{a=1}^{3}\mathrm{d}^4x_a\mathrm{d}^4\theta_a\Phi(x_1,\theta_1)\left[\frac{1}{\Box - m\overline{m}}\delta^8(z_2-z_1)\right]\left[\frac{1}{\Box - m\overline{m}}\delta^8(z_3-z_1)\right] \times$$
$$\times \left[\frac{-Q_2^2}{4}\frac{-D_2^2}{4}\frac{-\overline{D}_2^2}{4\Box}\frac{1}{\Box - m\overline{m}}\delta^8(z_2-z_3)\right]\left[\frac{-Q_2^2}{4}\frac{-\overline{D}_2^2}{4}\frac{1}{\Box - m\overline{m}}\delta^8(z_2-z_3)\right].$$

Now factor the fermionic delta functions separately so that we can apply eq. (4.3) to replace the actions of the differential operators in the last two bracketed factors by $\overline{\theta}_2^2\Box$ and 1, respectively. Then we find that the expression becomes

$$-4g^3|C|\overline{m}^4\int \mathrm{d}^4x_1\mathrm{d}^4x_2\mathrm{d}^4x_3\mathrm{d}^4\theta(\overline{\theta}\overline{\theta})\Phi(x_1,\theta)\left[\frac{1}{\Box - m\overline{m}}\delta^8(x_2-x_1)\right] \times$$
$$\times \left[\frac{1}{\Box - m\overline{m}}\delta^8(x_3-x_1)\right]\left[\frac{1}{\Box - m\overline{m}}\delta^8(x_2-x_3)\right]\left[\frac{1}{\Box - m\overline{m}}\delta^8(x_2-x_3)\right],$$

which confirms eq. (4.6).

4.4 A term without $\overline{\theta}\overline{\theta}$

We also obtain terms in the effective action with Q^2 but without $\overline{\theta}\overline{\theta}$. An example is shown in part (*a*) of figure 4, corresponding to the term $(Q^2\Phi)^2\Phi\overline{\Phi}$ from e.g. the four-point function $\langle F^3\overline{F}\rangle$. In this particular calculation, both factors of Q^2 end up on external lines after integration by parts. This contribution again diverges logarithmically.

4.5 'F-terms' are not holomorphic

As implied by the new (non)renormalization theorem, the F-term is no longer holomorphic. The second example (parts (*b*,*c*) of figure 4) shows a logarithmically divergent two-loop function contribution to $\langle FF\rangle$ that yields the term $\overline{\theta}\overline{\theta}(Q^2\Phi)\Phi$ in the effective action, as in subsection 4.1. Because this contribution involves the antiholomorphic coupling, $\overline{g}$, we see that terms that can *appear* like F-terms are not restricted to involve only holomorphic couplings. We can no longer distinguish F-terms and D-terms.

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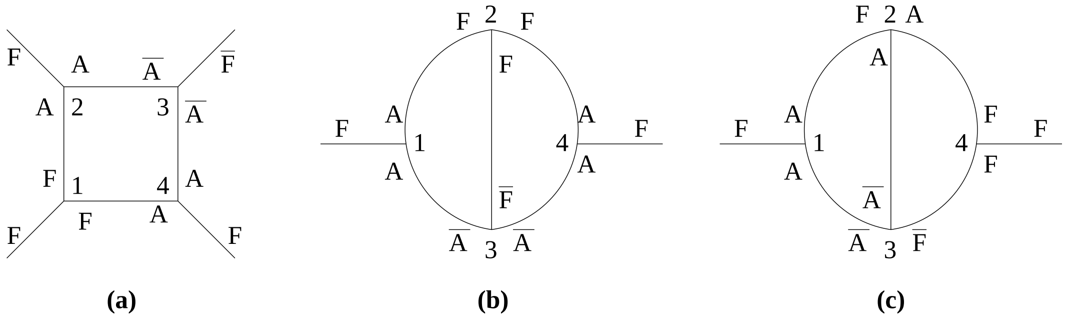

Figure 4: (a) The diagram without the generation of $\overline{\theta\theta}$ in the calculation of superfield computation. (b,c) The diagrams of a holomorphic part which depend on the antiholomorphic parameter $\overline{g}$.

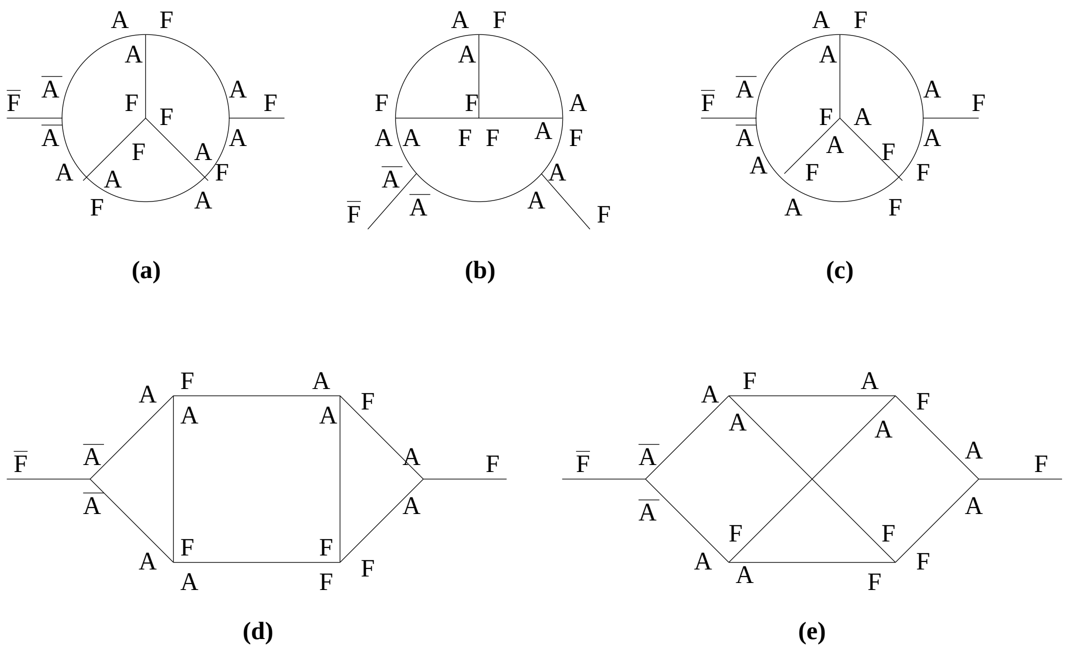

Figure 5: The three loop correction for $\langle F\overline{F}\rangle$ with one F^3 insertion. There are a lot of diagrams while we keep only five of them. The others cancel between bosonic and fermionic loop contributions. For these five remaining diagrams, there is no corresponding fermion loop contribution, and the result is finite.

4.6 A term with neither Q^2 nor $\overline{\theta\theta}$

Finally, the three-loop example drawn in figure 5 shows that we can generate terms with neither Q^2 nor $\overline{\theta\theta}$. This is a contribution of the the two-point function $\langle F\overline{F}\rangle$ to the term $\Phi\overline{\Phi}$.

5. Symmetry Considerations

5.1 Vanishing vacuum energy

For the deformed Wess-Zumino model, we can identify two global $U(1)$ (pseudo)symmetries

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	dim	$U(1)_R$	$U(1)_\Phi$		dim	$U(1)_R$	$U(1)_\Phi$
θ	$-1/2$	1	0	$\overline{\theta}$	$-1/2$	-1	0
$d\theta$	$1/2$	-1	0	$d\overline{\theta}$	$1/2$	1	0
A	1	1	1	$\overline{A}$	1	-1	-1
ψ	$3/2$	0	1	$\overline{\psi}$	$3/2$	0	-1
F	2	-1	1	$\overline{F}$	2	1	-1
g	0	-1	-3	$\overline{g}$	0	1	3
m	1	0	-2	$\overline{m}$	1	0	2
$C^{\alpha\beta}$	-1	2	0	$\|C\|$	-2	4	0
Φ	1	1	1	$\overline{\Phi}$	1	-1	-1

Table 1: Global charge assignments.

by treating all coupling parameters, including $C^{\alpha\beta}$, as the lowest components of (anti)chiral superfields. They are $U(1)_\Phi$ flavor symmetry and $U(1)_R$ R-symmetry. Charge assignment is given as table 1.

Let Λ be an ultraviolet cutoff scale. One can then construct a set of couplings of mass-dimension d, charge $q_R = R$ and $q_\Phi = S$ as

$$\Lambda^d \overline{g}^R \left(\frac{\overline{m}}{\Lambda}\right)^{\frac{S-3R}{2}} \left(g^4 \left(\frac{\overline{m}}{\Lambda}\right)^6 |C|\Lambda^2\right)^Z f\left(g\overline{g}, \frac{m\overline{m}}{\Lambda^2}\right). \tag{5.1}$$

Here $f(x,y)$ is an arbitrary function of x, y, and Z is a nonnegative integer. To show that the vacuum energy is zero, we consider the vacuum-to-vacuum amplitude directly:

$$\exp\left(-\int \mathrm{d}^4x E[C]\right) = \mathcal{Z} = \int \mathcal{D}[\Phi] \exp\left(-S_0 + \frac{g}{3}|C| \int \mathrm{d}^4x \left[\left(-\frac{Q^2}{4}\Phi\right)^2 \Phi\right]_{\theta^2}\right). \tag{5.2}$$

Here S_0 is the action of the ordinary Wess-Zumino model. To compute quantum corrections due to the deformation, consider the limit of small $C^{\alpha\beta}$, and expand the energy density perturbatively as

$$E[C] = \Lambda^4 \sum_{n=1}^{\infty} |C|^n \left(g^4 \overline{m}^6 \Lambda^{-4}\right)^n f_n\left(g\overline{g}, \frac{m\overline{m}}{\Lambda^2}\right), \tag{5.3}$$

where we have utilized the following observations: (1) the $|C|$-independent contribution ($n = 0$ term) is zero by the standard non-renormalization theorem, and (2) the vacuum energy is governed by the coupling eq. (5.1) with $d = 4$ and $R = S = 0$. Now take a partial derivative of equation (5.2) with respect to $|C|$, and then set $|C| = 0$, to get

$$\langle \mathrm{vac}| - \frac{g}{3}F^3 |\mathrm{vac}\rangle|_{C=0} = \Lambda^4 \left(g^4 \overline{m}^6 \Lambda^{-4}\right)^4 f_1\left(g\overline{g}, \frac{m\overline{m}}{\Lambda^2}\right). \tag{5.4}$$

The left-hand side is the vacuum expectation value of the operator F^3 in the ordinary Wess-Zumino model. However, since gF^3 has charges $q_R = -4$ and $q_\Phi = 0$, and $U(1)_R$ is the global (pseudo)symmetry in the ordinary Wess-Zumino model, the expectation value must be zero, so the function f_1 must vanish. By taking partial derivatives successively and repeating the same sort of argument, we can show that all of the f_n vanish. Thus the vacuum energy, as parametrized in equation (5.3), is zero.

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5.2 $\mathcal{N} = 1/2$ SUSY ground states

In ordinary $\mathcal{N} = 1$ supersymmetric theories, the supersymmetric vacua are critical points of the holomorphic superpotential $W(A)$ or, by conjugation, those of the antiholomorphic superpotential $\overline{W}(\overline{A})$. How about deformed, $\mathcal{N} = 1/2$ supersymmetric theories?

We now argue that the perturbative supersymmetric vacua still require

$$\overline{W}'(\overline{A}) = 0\,. \tag{5.5}$$

As the antiholomorphic superpotential $\overline{W}$ is not renormalized, these vacua are stable against radiative corrections.

To show eq. (5.5), it is sufficient to set all fermions to zero and consider bosonic fields that are constant in space. In this case, the tree-level part of the effective action takes the form

$$\Gamma = \int \mathrm{d}^4x [F\overline{F} + FW'(A) + \overline{F}\overline{W}'(\overline{A}) - \epsilon F^3]\,,$$

where ϵ is an abbreviation for $-g|C|/3$. Notice that the deformation term is proportional to F. Now consider radiatively generated terms in the effective action. Since $\partial_m = 0$ for constant bosonic fields, the operators Q_α, D_α, and $\overline{D}_{\dot{\alpha}}$, are simply partial derivatives with respect to θ^α or $\overline{\theta}^{\dot{\alpha}}$. So, after performing the $\mathrm{d}^2\theta$ integral in the effective action, every radiatively generated term must have at least one factor of F. Thus, the full effective action for constant bosonic fields is expressible in the form

$$\Gamma = \int \mathrm{d}^4x \left[F\overline{F} + FW'(A) + \overline{F}\overline{W}'(\overline{A}) + FK(A, \overline{A}, F, \overline{F})\right]\,, \tag{5.6}$$

where $K(A, \overline{A}, F, \overline{F})$ is a polynomial including both the tree-level deformation term and the radiatively generated F- and D-terms.

We now integrate out the auxiliary field $\overline{F}$. Its equation of motion from eq. (5.6) is

$$0 = F + \overline{W}'(\overline{A}) + F\frac{\partial K}{\partial \overline{F}}\,.$$

We see immediately that F is proportional to $\overline{W}'(\overline{A})$. Perturbatively, we can expand this proportionality factor $1/(1 + \frac{\partial K}{\partial \overline{F}})$ in powers of component fields, and use this to replace F in eq. (5.6). We readily see that the effective action is proportional to $\overline{W}'(\overline{A})$. This means that, perturbatively, the scalar potential is of the form

$$V = \overline{W}'(\overline{A})\Big[W'(A) - H(A, \overline{A})\Big]\,, \tag{5.7}$$

where $H(A, \overline{A})$ denotes the aforementioned perturbation. By solving $\frac{\partial V}{\partial A} = \frac{\partial V}{\partial \overline{A}} = 0$, we find that a set of the $\mathcal{N} = 1/2$ supersymmetric vacua with vanishing vacuum energy is given precisely by the critical points eq. (5.5) and $W'(A) - H(A, \overline{A}) = 0$.

Two remarks are in order. First, the scalar potential eq. (5.7) is in general complex-valued. This is expected: the deformation has introduced non-Hermiticity to the lagrangian. Second, as $H(A, \overline{A})$ is renormalized at each order in perturbation theory, solutions of $W'(A) - H(A, \overline{A}) = 0$ are not stable under radiative corrections.

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5.3 $\mathcal{N} = 1/2$ SUSY and antichiral rings

The fact that the vacuum energy vanishes in the deformed Wess-Zumino model leads to useful information concerning the vacuum state. Recall that, after the deformation, the resulting $\mathcal{N} = 1/2$ supersymmetry algebra is given by

$$\begin{aligned} \{Q_\alpha, Q_\beta\} &= 0 \\ \{Q_\alpha, \overline{Q}_{\dot\alpha}\} &= 2\sigma^m_{\alpha\dot\alpha} P_m \\ \left\{\overline{Q}_{\dot\alpha}, \overline{Q}_{\dot\beta}\right\} &= 4C^{(mn)}_{\dot\alpha\dot\beta} P_m P_n \,, \end{aligned} \tag{5.8}$$

where $C^{(mn)}_{\dot\alpha\dot\beta} \equiv C^{\alpha\beta}\sigma^m_{\alpha\dot\alpha}\sigma^n_{\beta\dot\beta}$. Taking vacuum expectation values, we get a nontrivial relation from the second line:

$$0 = \langle \mathrm{vac}|E|\mathrm{vac}\rangle = \left\langle \mathrm{vac}|Q_\alpha \overline{Q}_{\dot\alpha} + \overline{Q}_{\dot\alpha} Q_\alpha|\mathrm{vac}\right\rangle ,$$

because the vacuum energy vanishes. Since $\overline{Q}_{\dot\alpha}$ corresponds to the generator of explicitly broken supersymmetry (corresponding to translation of $\overline{\theta}$-coordinates), $\overline{Q}_{\dot\alpha}|\mathrm{vac}\rangle$ does not vanish in general. Therefore, we are led to conclude that

$$Q_\alpha|\mathrm{vac}\rangle = 0 \qquad \text{and} \qquad \langle \mathrm{vac}|Q_\alpha = 0 \tag{5.9}$$

for the $\mathcal{N} = 1/2$ supersymmetric vacuum.

As the theory is defined by a non-hermitean lagrangian, $|\mathrm{vac}\rangle$ and $\langle\mathrm{vac}|$ are not a priori related, but for $\mathcal{N} = 1/2$ supersymmetric vacuum, not only $|\mathrm{vac}\rangle$ but also $\langle\mathrm{vac}|$ is annihilated by Q_α. Moreover, as the theory is defined on Euclidean space, Q_α and $\overline{Q}_{\dot\alpha}$ are not hermitean conjugates, so the energy of a given state is not necessarily positive-definite. Rather, as discussed in the preceding subsection (see eq. (5.7)), it is in general complex-valued.

Along with the non-renormalization of the antiholomorphic superpotential, the relations eq. (5.9) play the crucial role for defining the antichiral ring. Recall that the antichiral ring can be defined as a set of operators $\overline{\mathcal{O}}$ obeying $\left[Q_\alpha, \overline{\mathcal{O}}\right\} = 0$. It then follows that $\overline{\mathcal{O}} \sim \overline{\mathcal{O}} + [Q_\alpha, X\}$ for every operator X, since

$$\begin{aligned} \left\langle \mathrm{vac}|\,[Q_\alpha, X\}\,\overline{\mathcal{O}}_2 \cdots \overline{\mathcal{O}}_n|\mathrm{vac}\right\rangle = &\left\langle \mathrm{vac}|Q_\alpha(X\overline{\mathcal{O}}_2 \cdots \overline{\mathcal{O}}_n)|\mathrm{vac}\right\rangle \pm \\ &\pm \left\langle \mathrm{vac}|X\left[Q_\alpha, \overline{\mathcal{O}}_2\right\} \cdots \overline{\mathcal{O}}_n|\mathrm{vac}\right\rangle \pm \cdots \pm \\ &\pm \left\langle \mathrm{vac}|X\overline{\mathcal{O}}_2 \cdots \left[Q_\alpha, \overline{\mathcal{O}}_n\right\}|\mathrm{vac}\right\rangle \pm \\ &\pm \left\langle \mathrm{vac}|(X\overline{\mathcal{O}}_1 \cdots \overline{\mathcal{O}}_n)Q_a|\mathrm{vac}\right\rangle . \end{aligned}$$

From the definition of antichiral operators, all terms except the first and the last ones vanish identically. It is here that the conditions eq. (5.9) come into play, ensuring these remaining two terms vanish as well. Likewise, to demonstrate that correlation functions of antichiral operators are independent of separations and they factorize, one needs to show that $\partial\overline{\mathcal{O}}$ vanishes inside the correlation functions. One again finds that, in proving this by using the eq. (5.8) relation $\partial\overline{\mathcal{O}} \sim \left[Q, \left[\overline{Q}, \overline{\mathcal{O}}\right\}\right\}$, the conditions eq. (5.9) play a crucial role.

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6. Further discussion

In this work, we have studied quantum aspects of $\mathcal{N}=1/2$ supersymmetric field theories in deformed superspace. We have found many intriguing and surprising features that warrant further investigation. Here we discuss two points we found interesting.

- The conclusions we have drawn in this work are not from any specific choice of the superpotential. In fact, for an arbitrary polynomial $W(\Phi)$ and $\overline{W}(\overline{\Phi})$, one readily finds that the lagrangian may still be recast as an ordinary Wess-Zumino model with a number of deformation terms. Explicitly, F- and $\overline{\text{F}}$-terms are given in a compact parametric form by

$$L_F = \mathcal{K}\Big(FW'(A) - \frac{1}{2}W''(A)\psi\psi\Big)\,, \qquad L_{\overline{F}} = \Big(\overline{F}\overline{W}'(\overline{A}) - \frac{1}{2}\overline{W}''(\overline{A})\overline{\psi}\overline{\psi}\Big)\,,$$

  where $\mathcal{K}$ is a functional differential operator:

$$\mathcal{K} \equiv \int_0^1 \mathrm{d}\tau \cos\Big[\tau|C|^{1/2}F\frac{\partial}{\partial A}\Big]\,.$$

  Generalization to N-component Wess-Zumino model is straightforward and replaces $F\partial_A$ by $F^a\partial_{A_a}$.

  The deformation part in the F-term contains operators of odd powers of F only, but, as evident from our analysis, operators of all powers of F are always generated radiatively. Large N expansion of this model might find interesting applications to various statistical mechanical systems.

- The radiatively generated F^2-term is interesting. A consequence would be that the term gives rise to mass splitting between the boson and fermion component fields. To illustrate this, consider the quadratic part of the lagrangian, including an ϵF^2-term, and integrate out the auxiliary fields, $F, \overline{F}$. The bosonic part of the lagrangian becomes

$$\overline{A}\Box A - m\overline{m}A\overline{A} + \epsilon\overline{m}^2\overline{A}^2\,.$$

  If we decompose field A into the real basis $A = a + ib$ we get

$$a\Box a + b\Box b - (|m|^2 - \epsilon\overline{m}^2)a^2 - (|m|^2 + \epsilon\overline{m}^2)b^2 - 2i\epsilon\overline{m}^2 ab\,,$$

  while the fermionic part of the lagrangian is unaffected. We see that the mass of the two real bosons are split by $\pm\epsilon\overline{m}^2$ and that the two real bosons mix, analogous to the $K^0 - \overline{K}^0$ mesons, albeit the theory is defined in Euclidean space.

The possibility that the deformation-induced ϵF^2 term gives rise to mass splitting and flavor oscillation, while keeping the vacuum energy to zero, might find interesting phenomenological applications, once a suitable deformation can be achieved for lorentzian superspace.

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Acknowledgments

We are grateful to F. Cachazo, O. Lunin, and N. Seiberg for numerous helpful discussions. B.F. would like to thank V. Balasubramanian, A. Naqvi, Y.H. He and V. Jejjala for discussions. R.B. and B.F. were supported by the NSF grant PHY-0070928. S.J.R. was supported in part by the KOSEF Interdisciplinary Grant 98-07-02-07-01-5 and the KOSEF Leading Scientist Grant. S.J.R. was a Member at the Institute for Advanced Study during this work. He thanks the School of Natural Sciences for hospitality and for the grant in aid from the Fund for Natural Sciences.

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